Homework 2 (solution)

1.

1.1

Final value

$$F(s) = \frac{1}{s(s+2)^2}$$

$\lim_{t\to\infty} f(t) = \lim_{s\to 0}[sF(s)]$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0}\left(\frac{\cancel{s}\cdot 1}{\cancel{s}(s+2)^2}\right) \to \lim_{s\to 0}\frac{1}{(s+2)^2} = \frac{1}{4}$$

1.2

$$\frac{a}{s} + \frac{b}{s+2} + \frac{c}{(s+2)^2} = \frac{1}{s(s+2)^2}$$

Inverse transformation

from partial fraction

$$F(s) = \overset{a}{\frac{1}{4s}} - \overset{b}{\frac{1}{4}}\frac{1}{s+2} - \overset{c}{\frac{1}{2}}\frac{1}{(s+2)^2}$$

see slide 21, line 6.

$$\boxed{f(t) = \frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}te^{-2t}}$$

2.

2.1

Assuming all initial conditions are zero, determine the solution of the following differential equation, where the forcing term is $\boxed{f(t) = 2e^{-t}.}$

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 2y(t) = f(t)$$

$$\mathcal{L} = \frac{2}{s+1}$$

$$(s^3 + 4s^2 + 5s + 2)\,Y(s) = \frac{2}{s+1}$$

$$Y(s) = \frac{2}{(s+1)(s^3+4s^2+5s+2)} = \frac{2}{(s+1)^3(s+2)}$$

roots $-1, -1, -2$

partial fraction

$$Y(s) = \frac{a}{(s+1)^3} + \frac{b}{(s+1)^2} + \frac{c}{(s+1)} + \frac{d}{s+2}$$

$a = 2$

$b = -2$

$c = 2$

$d = -2$

partial fraction

$$Y(s) = \frac{2}{(s+1)^3} - \frac{2}{(s+1)^2} + \frac{2}{s+1} - \frac{2}{s+2}$$

$$y(t) = t^2 e^{-t} - 2te^{-t} + 2e^{-2t} - 2e^{-2t}$$